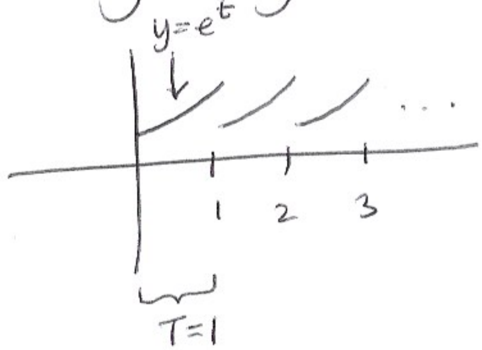
$$y'' + 3y' + 2y = f(t) \qquad y(0) = y'(0) = 0$$



$$f_T(t) = \begin{cases} e^t, & 0<t<1 \\ 0, & t>1 \end{cases} = e^t + u(t-1)(0-e^t)$$

$$= e^t - u(t-1)\underbrace{e^t}$$

$$e^{t+1} = e^t$$

$$= e \cdot e^t$$

$$\mathcal{L}\{f_T\} = \frac{1}{s-1} - e^{-s}\frac{e}{s-1}$$

$$F = \mathcal{L}\{f\} = \frac{\mathcal{L}\{f_T\}}{1-e^{-sT}} = \frac{\frac{1}{s-1} - e^{-s}\frac{e}{s-1}}{1-e^{-s}}$$

$$s^2Y - \cancel{sy(0)} - \cancel{y'(0)} + 3(sY - \cancel{y(0)}) + 2Y = F$$

$$s^2Y + 3sY + 2Y = (s^2+3s+2)Y = F$$

$$Y = \frac{F}{s^2+3s+2} = \frac{1}{1-e^{-s}} \cdot \frac{1}{(s-1)(s+1)(s+2)} - \frac{e^{-s}}{1-e^{-s}}\, e\, \frac{1}{(s-1)(s+1)(s+2)}$$

$$s^2+3s+2 = (s+1)(s+2)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

INFINITE GEOMETRIC SERIES

$$S = \frac{a_1}{1-r} \qquad a_1 = 1,\ r = x$$

$$\frac{1}{1-e^{-s}} = 1 + e^{-s} + e^{-2s} + e^{-3s} + \cdots$$

$$\frac{e^{-s}}{1-e^{-s}} = e^{-s} + e^{-2s} + e^{-3s} + \cdots$$

$$Y = \left(1 + e^{-s} + e^{-2s} + \cdots\right)\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$- \left(e^{-s} + e^{-2s} + \cdots\right) e \left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$Y = \cancel{\frac{1}{6}e^{t} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t}} \quad \frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}$$

$$+ \left(e^{-s} + e^{-2s} + e^{-3s} + \cdots\right)(1-e)\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$y = \frac{1}{6}e^{t} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t}$$

$$+ (1-e)\sum_{n=1}^{\infty} u(t-n)\left(\frac{1}{6}e^{t-n} - \frac{1}{2}e^{-t+n} + \frac{1}{3}e^{-2t+2n}\right)$$

$$\frac{1}{(s-1)(s+1)(s+2)} = \frac{\overset{\frac{1}{6}}{\cancel{A}}}{s-1} + \frac{\overset{-\frac{1}{2}}{\cancel{B}}}{s+\ldots}$$

$$1 = A(s+1)(s+2) + B(s-\ldots$$

$$+ C(s-1)\ldots$$

$s=-1: \ 1 = B(-2)(1) \to B = \ldots$

$s=-2: \ 1 = C(-3)(-1) \to C = \ldots$

$s=1: \ 1 = A(2)(3) \to A = \ldots$

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$$s=-3: \ \frac{1}{(-4)(-2)(-1)} = -\frac{1}{8}$$

$$\frac{\frac{1}{6}}{-4} + \frac{-\frac{1}{2}}{-2} + \frac{\frac{1}{3}}{-1} = -\frac{1}{24}\ldots$$

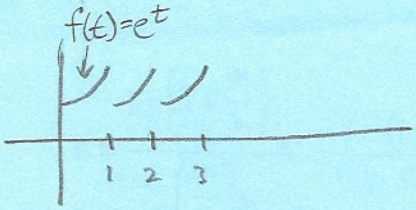
$$= \frac{-1+6-8}{24}\ldots$$

$$\mathcal{L}^{-1}\left\{e^{-ns}\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1}\ldots\right.\right.$$

$$= u(t-n)\left(\frac{1}{6}e^{t-n} - \frac{1}{2}e^{-t\ldots}\right.$$

$$+ \frac{1}{3}e^{-2\ldots}$$

$$y'' + 3y' + 2y = f(t) \qquad y(0) = y'(0) = 0$$



$$f_T(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & t > 1 \end{cases} = e^t - u(t-1)e^t \qquad e^{t+1} = e \cdot e^t$$

$$\mathcal{L}\{f_T\} = \frac{1}{s-1} - e \cdot e^{-s}\frac{1}{s-1}$$

$$\mathcal{L}\{f\} = \frac{\frac{1}{s-1} - e \cdot e^{-s}\frac{1}{s-1}}{1-e^{-s}} = F$$

$$s^2Y - s\cancel{y(0)} - \cancel{y'(0)} + 3(sY - \cancel{y(0)}) + 2Y = F$$

$$Y = \frac{F}{s^2+3s+2} = \frac{1}{1-e^{-s}} \frac{1}{(s-1)(s+1)(s+2)} - \frac{e^{-s}}{1-e^{-s}} \cdot e \cdot \frac{1}{(s-1)(s+1)(s+2)}$$

$$\frac{1}{1-e^{-s}} = 1 + e^{-s} + e^{-2s} + e^{-3s} + \cdots$$

$$\frac{e^{-s}}{1-e^{-s}} = e^{-s} + e^{-2s} + e^{-3s} + \cdots$$

$$\frac{1}{(s-1)(s+1)(s+2)} = \frac{\overset{\frac{1}{6}}{A}}{s-1} + \frac{\overset{-\frac{1}{2}}{B}}{s+1} + \frac{\overset{\frac{1}{3}}{C}}{s+2} \qquad s = 2 \quad \frac{1}{1(3)(4)} = \frac{1}{12}$$

$$\frac{\frac{1}{6}}{1} - \frac{\frac{1}{2}}{3} + \frac{\frac{1}{3}}{4} = \cancel{\frac{1}{6} - \frac{1}{6}} + \frac{1}{12} \checkmark$$

$$Y = (1 + e^{-s} + e^{-2s} + e^{-3s} + \cdots)\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$-(e^{-s} + e^{-2s} + e^{-3s} + \cdots)(e)\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$= \frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2} + (e^{-s} + e^{-2s} + e^{-3s} + \cdots)(1-e)\left(\frac{\frac{1}{6}}{s-1} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{3}}{s+2}\right)$$

$$= \frac{1}{6}e^t - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} + \sum_{n=1}^{\infty} u(t-n)(1-e)\left(\frac{1}{6}e^{t-n} - \frac{1}{2}e^{-t+n} + \frac{1}{3}e^{-2t+2n}\right)$$

$$= \sum_{n=0}^{\infty} u(t-n)(1-e)\left(\frac{1}{6}e^{t-n} - \frac{1}{2}e^{-t+n} + \frac{1}{3}e^{-2t+2n}\right)$$

$$= \left(\frac{1}{6}e^t - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t}\right) + u(t-1)(1-e)\left(\frac{1}{6}e^{t-1} - \frac{1}{2}e^{-t+1} + \frac{1}{3}e^{-2t+2}\right)$$

$$\cdots \, e^{t-2} \, \cdots \, e^{-t+2} \, \cdots \, e^{-2t+4})$$

$$= \frac{1}{6}e^{t} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} \qquad \text{IF } 0<t<1$$

$$= \frac{1}{6}e^{t} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} + (1-e)\left(\frac{1}{6}e^{t-1} - \frac{1}{2}e^{-t+1} + \frac{1}{3}e^{-2t+2}\right)$$

$$= \cancel{\frac{1}{6}e^{t}} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} + \frac{1}{6}e^{t-1} - \cancel{\frac{1}{6}e^{t}} - \frac{1}{2}e^{-t+1} + \frac{1}{2}e^{-t+2} + \frac{1}{3}e^{-2t+2} - \frac{1}{3}e^{-2t+3}$$

$$= \frac{1}{6}e^{t-1} - \frac{1}{2}e^{-t}(1+e-e^{2}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3})$$

$$= 1 + e^{2}(1-e)$$

$$= e + \frac{(1-e)((e^{2})^{2}-1)}{e^{2}-1}$$

$$\text{IF } 1<t<2$$

$$= \frac{1}{6}e^{t-1} - \frac{1}{2}e^{-t}(1+e-e^{2}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}) + (1-e)\left(\frac{1}{6}e^{t-2} - \frac{1}{2}e^{-t+2} + \frac{1}{3}e^{-2t+4}\right)$$

$$= \frac{1}{6}e^{t-2}(e+1-e) - \frac{1}{2}e^{-t}(1+e-e^{2}+(1-e)e^{2}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}+(1-e)e^{4})$$

$$= \frac{1}{6}e^{t-2} - \frac{1}{2}e^{-t}(1+e-e^{3}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}+e^{4}-e^{5})$$

$$= 1 + e^{2}(1-e) + e^{4}(1-e)$$

$$= e + \frac{(1-e)((e^{2})^{3}-1)}{e^{2}-1}$$

$$\text{IF } 2<t<3$$

$$= \frac{1}{6}e^{t-2} - \frac{1}{2}e^{-t}(1+e-e^{3}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}+e^{4}-e^{5}) + (1-e)\left(\frac{1}{6}e^{t-3} - \frac{1}{2}e^{-t+3} + \frac{1}{3}e^{-2t+6}\right)$$

$$= \frac{1}{6}e^{t-3}(e+1-e) - \frac{1}{2}e^{-t}(1+e-e^{3}+(1-e)e^{3}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}+e^{4}-e^{5}+(1-e)e^{6})$$

$$= \frac{1}{6}e^{t-3} - \frac{1}{2}e^{-t}(1+e-e^{4}) + \frac{1}{3}e^{-2t}(1+e^{2}-e^{3}+e^{4}-e^{5}+e^{6}-e^{7})$$

$$\downarrow$$

$$1 + e^{2}(1-e) + e^{4}(1-e) + e^{6}(1-e)$$

$$= e + \left[(1-e) + e^{2}(1-e) + e^{4}(1-e) + e^{6}(1-e)\right]$$

$$= e + \frac{(1-e)((e^{2})^{4}-1)}{e^{2}-1}$$

$$= e - \frac{(e^{2})^{4}-1}{e+1}$$

$$\text{IF } 3<t<4$$

6.1 SERIES SOLUTIONS

POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$

"CENTERED ABOUT $x=c$"

DEFINE $0^0 = 1$ FOR POWER SERIES

$$f'(x) = \frac{d}{dx} \sum_{n=0}^{\infty} a_n (x-c)^n = \sum_{n=0}^{\infty} \frac{d}{dx} a_n (x-c)^n$$

$$= \sum_{n=\cancel{0}\,1}^{\infty} n a_n (x-c)^{n-1}$$

IF $n=0$, TERM $=0$

LET $m = n-1 \rightarrow n = m$

$n=1 \rightarrow m = 1-1 =$

$n=\infty \rightarrow m=\infty$

$$\sum_{m=0}^{\infty} (m+1) a$$

$(x-c)^m$

$$= \sum_{n=0}^{\infty} (n+1) a_{n+1} ($$

$$f''(x) = \frac{d}{dx} f'(x) = \frac{d}{dx} \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}$$

$$= \sum_{n=\cancel{1}\,2}^{\infty} n(n-1) a_n (x-c)^{n-2}$$

IF $n=1$, TERM $=0$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} (x-c)^n$$

SHIFT DOWN 2

SHIFT UP 2

INDEX SHIFT

SHIFT FUNCTION UP BY k

SHIFT LIMITS DOWN BY k